Bitcoin and its offspring: a volatility risk approach

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Abstract

I examine the risk-return relationship between the return on Bitcoin and the returns on its forks (Litecoin, Bitcoin Cash, Bitcoin Gold, Bitcoin Diamond, and Bitcoin Private). Forks often create price volatility and instability, and I provide evidence that there is a transmission of volatility risk from Bitcoin forks to Bitcoin. The three multivariate volatility models (EWMA, DCC, and BEKK) provide a better estimation of the conditional volatility and the time-varying correlation of Bitcoin with its forks than univariate volatility models. In particular, the time-varying correlation is negative during times of high volatility, but strong and positive in low volatility episodes. Therefore, Bitcoin and its forks behave as crypto-currencies during bad times and as assets during good times. The strong and positive correlation for most of the sample period indicates that Bitcoin forks do not offer a hedge against Bitcoin risk.

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1 Introduction

Since the creation of Bitcoin in October 2008, by Satoshi Nakamoto[1], many new crypto-currencies have been created on top of Bitcoin technology (protocol). The proof-of-work is the Bitcoin consensus protocol which is open source, thus there is a natural incentive to copy this technology, or modify it and create a new peer-to-peer network. The latter is known as a fork. Generally speaking, a fork is a change in the set of rules of the original software (original blockchain) to develop a new version of it (new blockchain). Similar to Narayanan et al. (2016) and Antonopoulos (2017), I identify three main factors for a fork on the Bitcoin technology (details will be discussed in the next section). The first factor is the block size to store information. Bitcoin is a peer-to-peer decentralized and distributed public ledger, and each block of this ledger contains a record of all Bitcoin transactions. As Bitcoin became popular, the number of transactions increased and with it the problem of storing this information in a block. The restriction of storing this information leads us to the second factor: high transaction fees. The creator of any transaction has to pay a fee to a miner[2] in order to add her transaction into the block. Due to the fact that the block size is fixed and the number of transactions has increased over time, the creator of a transaction pays higher fees as an incentive for a miner to prioritize and add her transaction into the block. The third factor is mining centralization. That is, when a small number of miners or pool of miners control most of the computational power to process Bitcoin transactions.

As a result, Bitcoin blockchain has become a slow, expensive, and centralized payment system. To fix this problem, the Bitcoin community would need to adopt a new approach to establishing agreement on what transactions are valid in the ledger. As the Bitcoin technology stands at present, the only way to solve this problem is by a fork.

Forks often create price volatility and increase uncertainty in the market, but its implications are not fully understood. This paper tries to fill this gap by examining the relationship between the return on Bitcoin and the returns on its forks. I select the five major Bitcoin forks based on market capitalization and data availability: Litecoin (October 2011), Bitcoin Cash (August 2017), Bitcoin Gold (October 2017), Bitcoin Diamond (November 2017), and Bitcoin Private (February 2018). I aim to provide an understanding of the transmission of the volatility risk after a fork occurs. The volatility risk transmission from a fork to Bitcoin could be direct through its conditional variance or indirect through its conditional covariances.

To that end, I proceed in two stages. In the first stage, I obtain the time-varying correlation based on univariate GARCH models. Earlier studies have used univariate GARCH models to calculate the volatility but not the dynamic correlation among virtual currency returns. Bouoiyour and Selmi (2015) compare two periods of Bitcoin volatility by estimating EGARCH and TGARCH models, whereas Dyhrberg (2016) uses an EGARCH to study the capabilities of Bitcoin in terms of risk management. Using several volatility models, Katsiampa (2017) makes a comparison of GARCH specifications for modeling Bitcoin volatility, and Chu et al. (2017) use twelve GARCH models to analyze the volatility of seven crypto-currencies; just to name a few.

[1]Satoshi Nakamoto is the nickname used by the person or persons who wrote Bitcoin white paper, and created Bitcoin protocol.

[2]As discussed in the next section, a user is not necessarily a miner.

In the second stage, as a robustness check, I consider a multivariate volatility approach by using Exponentially Weighted Moving Average (EWMA), DCC-GARCH and BEKK-GARCH models. The benefit of the BEKK-GARCH method is that it allows dependence between the volatility series, something that cannot be done with univariate volatility models or with the other multivariate GARCH specifications (Bauwens et al., 2006). There are few studies which have used a multivariate GARCH methodology to study simultaneously the variances and covariances of crypto-currency returns. For instance, Bouri et al. (2017) employ a DCC-GARCH to claim that Bitcoin can act as a hedge for equity indices, bonds, oil, and gold. Likewise, Corbet et al. (2018) were among the first to measure the interrelation between the crypto-currency returns, but they only study Bitcoin, Ripple, and Litecoin. Beneki et al. (2019) apply the BEKK-GARCH methodology to investigate the volatility spillovers between Bitcoin and Ethereuem.

The data used are the daily closing prices for Bitcoin, Litecoin, Bitcoin Cash, Bitcoin Gold, Bitcoin Diamond, and Bitcoin Private from April 28, 2013 (as the earliest date available for the other Bitcoin forks) to August 31, 2019. I think that the sample period is relevant because it includes the year 2017. During this year, Bitcoin pushed the market into a bubble (Corbet et al., 2018; Beneki et al., 2019), with prices reaching their peak in December 2017[3], followed by a sharp decrease. Also, three Bitcoin forks (Bitcoin Cash, Bitcoin Gold, and Bitcoin Diamond) began to unfold in October 2017.

In this research, I contribute to existing literature in three ways. First, after explaining the economic factors behind Bitcoin forks, I find that 1) forks since 2017 were driven by the excess demand for storing Bitcoin transactions in a block, the highest transaction fees during the bubble period, and the fact that the block size hit the limit of 1MB during the bubble period and remained around 0.9MB thereafter; and 2) mining centralization evolved like a market entry dynamic game (huge investment in hardware as a barrier to entry the market). Second, I estimate in a novel way the time-varying correlation of each pair of crypto-currency returns (Bitcoin and each of its forks) using univariate volatility models, absent in previous literature. This approach was recently applied to the forex market but not to the crypto-currency market (Bazán-Palomino, 2019). Third, I provide a robustness check of the estimated time-varying correlation by applying three multivariate GARCH models to each pair of crypto-currency returns. This is the first study to apply both univariate and multivariate volatility approaches to a particular consensus protocol (proof-of-work) and its forks; to the best of my knowledge.

If the forks are sharing the same technology, we would expect substitutability among crypto-currencies. My univariate and multivariate results show a negative correlation for the last two months of 2017, indicating that there was a substitutability among crypto-currencies during high risk times. However, the correlation became positive for the rest of the sample, suggesting that they behave more like stocks rather than currencies.

Regarding the multivariate approach, all volatility models produce higher correlations than my univariate

[3]Bitcoin went from USD 998.33 on January 1, 2017, to USD 19,497.4 (maximum historical price) on December 16, 2017. Over this year, Litecoin hit a record high price of USD 358.34 on December 18, 2017, representing a return of 7845.4% year-to-date. Bitcoin Cash started a precipitous rise shortly after its launch in August 2017, increasing from about USD 300 to a peak of USD 3,923 on December 20, 2017. Likewise, Bitcoin Gold was created in October 2017 and since then, its price meteorically raised from USD 142.92 to USD 453.45 on December 20, 2017, being its maximum level.

results. Applying univariate volatility models to calculate the time-varying correlation between crypto-currency returns could lead to underestimating the mutual impact between Bitcoin and its forks. Also, the BEKK-GARCH(1,1) offers more accurate modeling of the time-varying volatility and correlation which are of great importance for risk management and asset pricing. Since this method allows dependence between the volatility risk of crypto-currency returns, we can conclude that the volatilities of Bitcoin and each of its forks are dynamically related and this relationship is strong. Therefore, Bitcoin forks do not offer a hedge for Bitcoin risk.

The rest of the paper is organized as follows. Section 2 provides precise descriptions of the Bitcoin network and the drivers behind Bitcoin forks. Section 3 shows the data and the univariate and multivariate volatility models I use to estimate both the volatilities of return on Bitcoin and returns on its forks and the correlation between them. I present the results and discuss the main findings in Section 4. Finally, Section 5 concludes my arguments.

2 The Bitcoin network and the drivers behind Bitcoin forks

In this section, I describe what a blockchain is, how the Bitcoin consensus protocol works, and the factors that caused and would cause Bitcoin forks[4].

A blockchain is a public distributed ledger or a collection of transactions which is maintained by a network of users or computers, called nodes. Each of the nodes can store a copy of the blockchain, verify the authenticity of the blocks containing the transactions, and propose a new block. Here I make a distinction between a user and a miner. A user is a node or computer that can verify all transactions since the beginning and can do it in the future. On the other hand, a miner is a user with an additional feature, she can create a new block.

An important aspect of blockchain technology and, in particular, of Bitcoin is the consensus protocol or consensus mechanism. The main idea behind the consensus mechanism is how nodes agree to validate transactions and produce a new block of information (a set of new transactions) which is added to the chain. At every point in time, the majority of participants – 51% of the users in most of the crypto-currency networks – must agree about the ownership rights to the tokens or coins. Because of network participants can remain anonymous, the consensus mechanism is the core for the functionality of any virtual currency.

Proof-of-work is the Bitcoin consensus algorithm for verifying transactions. Under this protocol[5], miners compete against each other in a race for the right to add a new block of information (transactions) to the chain. To do so, miners have to solve a mathematical puzzle called the hash puzzle or hash algorithm. The winner reports both the new block and the solution of the hash puzzle, getting as a reward new Bitcoins and

[4]The source of information was crypto-currencies white papers and websites, and other online sources such as coinmarketcap.com, cointelegraph.com, coindesk.com, and coinbase.com.

[5]Antonopoulos (2017) mentions that Bitcoin consensus is based on four processes: a) independent verification of each transaction by any node, b) independent aggregation of Bitcoin transactions into a new block by a miner, c) independent verification of a new block by all users in the network, and d) independent selection of the chain with the highest proof-of-work (highest cumulative computational power) by any node.

transaction fees included in the block. This process is also known as mining, and in this way, new Bitcoins are added to the money supply. The role of miners is crucial since they are responsible for securing and confirming transactions by adding blocks of transaction information to the chain. Unlike fiat money, Bitcoin does not depend on a central authority providing a clearinghouse service or a central bank that controls the money supply.

In a general sense, Bitcoins are just created from nowhere. The transactions in each block are messages to transfer coins from one address to another. These transactions are broadcast to the network, and the first miner who puts together new transactions in a new block and solves a mathematical puzzle gets new Bitcoins and transaction fees.

Having explained what Bitcoin is and how it works, let's talk about the underlying factors of Bitcoin forks. The first factor – and perhaps the most important – is the block size to store information. From its inception, Bitcoin has a block size of 1MB[6], limiting the amount and frequency of transactions that the network can process. The original consensus protocol allows 2 to 7 transactions per second and a block time of 10 minutes. With its increasing popularity over time, given the current technology, it is very difficult for Bitcoin to meet demand.

The excess of demand for Bitcoin technology leads us to the second cause of a fork: high transaction fees. In order to have her transaction processed by a miner, the creator of a transaction has to pay a transaction fee. In this way, miners are compensated for the services they provide. Over the years, the number of users sending transactions increased, but only a limited number of transactions could be added to the chain. Therefore, there is congestion in the available block space and there are pending transactions. If the creator of a transaction wants to move her transaction to the top of the list, she has to pay a higher transaction fee to incentivize the miner to add it. The creators of transactions know that to have priority, they have to offer a high transaction fee to a miner. Consequently, the cost for getting a transaction into the next Bitcoin block rises.

Panel (a) of Figure 1 presents the Block Size which refers to data sent to legacy nodes. The Block Size hit the limit of 1MB during March 2016 - December 2017. The following three months, it remained around 0.5MB and ever since the data per block was above 0.9MB on average. Similarly, Panel (b) of Figure 1 shows the Mempool or memory pool which is the number of unconfirmed transactions that have been broadcast to the Bitcoin network. Over the crypto-currency bubble year, the mempool reached four main peaks: February (100,125), May (184,101), November (161,450), and December (184,106). In simple words, this is a measure of excess demand for storing Bitcoin transactions in a block.

This excess demand caused an adjustment in the hash algorithm. The Bitcoin network adjusts the hash algorithm every 2016 blocks based on a target time of 10 minutes per block. Valid blocks must have a hash below this target and if this target is adjusted, it is more difficult to find a block below the new hash target. As a result, more hash 'tries" are needed to have the same probability of finding a block which in turn

[6]The block size was arbitrarily set to 1MB and the reason was to prevent attack from hackers to the network.

brings more chances of mining profitability. Hash Rate is the estimated number of tera hashes per second which means how many mathematical calculations a miner performs per second (Panel (c) of Figure 1). This variable is a measure of mining difficulty. As explained before, the congestion in a block leads to an increase in the transaction fees (Panel (d) of Figure 1). The daily average Bitcoin transaction fee started to rise in February 2017 and reached its peak on December 21, 2017, 5 days after the highest Bitcoin price.

Before moving on to the next problem, centralization, it is necessary to talk a little more about the consensus protocol. According to Narayanan et al. (2016), the Bitcoin community achieves proof-of-work using hash puzzles, and the nodes which propose a new block are selected based on their computing power. Bitcoin hash puzzle is SHA-256 which is a cryptographic hash function used for mining. The hash function could be adjusted by the network in order to keep the average time between successive blocks within 10 minutes.

As Bitcoin has become mainstream, the hash puzzle has become more complex. The increasing difficulty of the mathematical puzzle causes centralization. That is, more transactions in the network lead to a more difficult hash puzzle, and a more complicated puzzle leads to centralization because only a few miners can invest in more powerful hardware and electricity. By doing so, a miner increases her probability to be the next proposer of a block. As a result, small miners have left the market and those that remain have worked together to increase their likelihood of finding a block to offset their mining cost. They have congregated into mining pools, i.e., they pool their hashing power and split their rewards proportionally to the amount of work they have done (Antonopoulos, 2017).

Figure 2 presents the biggest mining pools by the end of 2012 (left panel) and by the end of 2018 (right panel). In December 2012, more than 50% of the miners were unknown and small, and BTC Guild and SlushPool were the only miners with more than 10% of the mining hash power. Seven years later, the story is quite different. Five miners – BTC.com (17.42%), AntPool (12.37%), SlushPool (10.79%), ViaBTC (9.67%), and F2Pool (8.71%) – control 69.75% of the mining hash power. Unknown (6.45%) and other (11%) miners only represent 17.45% of the mining hash. In addition, the vast majority of miners who were active in 2012 are no longer in 2018. Only SlushPool remains active with its market participation steady at around 10%. An implication of the mining centralization is the possibility that the biggest five miners could cooperate to control the Bitcoin network influencing the transaction fees, hash rate, and Bitcoin price. Along these lines, Hayes (2017) found that the level of competition in the network of producers could explain the value of Bitcoin.

To save money on energy, miners moved to countries where the cost of energy is cheaper[7]. By the end of 2018, 81% of the network hash rate was concentrated in China, followed by the Czech Republic (10%), Iceland (2%), Japan (2%), Georgia (2%), and Russia (1%). Regarding the top miners in the network, BTC.com, Antpool, F2pool, ViaBTC, BTC.top, DPOOL, and 58COIN are located in China, while Slushpool is in the Czech Republic.

[7] According to the Bitcoin Energy Consumption Index (https://digiconomist.net/bitcoin-energy-consumption), the global energy consumption of all Bitcoin mining is equivalent to the power usage of the Czech Republic.

In summary, the constant block size, the increasing transaction fees and the centralization of miners have caused disagreements among Bitcoin users. In particular, the disagreements among miners have fostered incentives to create alternative crypto-currencies. Thus, we are at the forefront of a fork.

2.1 The Mining Game and Bitcoin forks

Inevitably, a hard fork is the solution to Bitcoin protocol problems. For the purpose of this paper, I consider a hard fork to happen whenever there is a change in the core Bitcoin protocol which causes two candidate blocks competing to form the longest blockchain. In simple words, it is a change in the set of rules of the original software (original blockchain) to develop a new version of it (new blockchain). Every time there is a Bitcoin fork, the miners have to decide whether or not to continue supporting the original blockchain. Under proof-of-work, if a miner wants to support both the new and the original blockchain, she has to split her computational power between the two.

As an economic principle, if Bitcoin miners are rational, they will act "honestly" as long as the benefits exceed the costs. That is, if block rewards and transaction fees are higher than electricity and hardware costs, broadly speaking, then it is beneficial to be a miner. The behavior of each miner is a Nash Equilibrium and she will follow the rules of this game – the consensus mechanism – as long as there is no incentive to deviate from the equilibrium.

However, the incentives have changed over time. Note that a miner is like any other firm in the market, it has revenues and production costs and the miner's primary objective is to maximize profits. On the revenues side, the market value of the reward is denominated in Bitcoins, and due to the enormous increase in Bitcoin price, the reward does not seem to be a problem. On the cost side, there are a couple of complications. First, the variable costs (electricity) and the fixed costs (computers) are denominated in fiat money, for example, in U.S. Dollars. Thus, there is a balance sheet effect in terms of the exchange rate between Bitcoin and the U.S. Dollar. Second, the computers are the fixed costs for a miner; it is like investing in a new factory. But a miner's level of investment depends on the investment of its competitors. If a miner wants to stay in the market, i.e., wants to make profits, she has to invest systematically in more computer power. This market environment is closely related to market entry dynamic games, where the new player (new miner) desires to enter and the incumbent (the existing miner) threatens the entrant with a huge investment in a factory.

How does it evolve? If a miner has the most powerful computer in the network, the rest of the users would complicate the mathematical puzzle and in this way, try to keep even the probability to be the next proposer of a new block. This change in the parameters of the hash puzzle will demand more computational power. Like in any other equilibrium, if a miner incurs losses, she will leave the market. On the contrary, if a miner makes a lot of profits, then she could invest in more powerful computers and therefore discourage new miners from entering the market. A natural consequence is that the difficulty of the hash puzzle will increase (Panel (c) of Figure 1), but only the miners who can afford new computers will stay in the market.

At the beginning, the variable costs (energy) did not constitute any immediate threat to current or new miners. However, this electricity cost is not negligible because more computational power leads to higher consumption of energy.

But the story does not end here. The blockchain industry is a capital-intensive industry. After a while, miners who stay in the network can act strategically. Since the capital investment is so high, the lead miners would expect to earn an attractive return on their investment. If they produce (mining Bitcoin) at a low level, the Bitcoin price and transaction fees stay high protecting their long-run competitive position. Unfortunately, due to a lack of information, it is not possible to study the direct connection between mining and Bitcoin price[8]. Also, studying the determinants of Bitcoin price is not within the scope of this document. I only present the economic incentives that cause a fork.

Returning to the rules of the game, the consensus protocol must change in order to correct the disequilibrium in the market. One way that can be done is if users agree to change the rules such as changing the hash algorithm or increasing the block size. The latter is more complicated than it seems and would not solve the problem of centralization. Under proof-of-work, a bigger block means a higher computational power, and, as discussed before, a higher computational power means better and more expensive mining hardware. Another way to fix the disequilibrium is to "copy " [9] the Bitcoin technology to meet the demand, causing a fork.

Bitcoin (BTC) has experienced five major forks causing instability to the value of crypto-currencies.

- *Litecoin (LTC):* In October 2011, a former Google engineer Charles Lee, created a Bitcoin clone which is considered a fork. Litecoin is based on Bitcoin protocol but differs in terms of the hashing algorithm (scrypt, instead of SHA-256), the total number of coins, and the time to generate a block. At the time of writing this paper, according to CoinMarketCap and CoinBase, Litecoin reduced the time to generate a block from 10 minutes to 2.5 minutes. The fork also lowered the transaction fees.

- *Bitcoin Cash (BCH):* On August 1, 2017, the Bitcoin network finally agreed to update the protocol and increase the block size to 8MB. This network agreement brought Bitcoin Cash (BCH) to life. According to Bitcoin website, the blockchain forked at block 478,558 and all Bitcoin holders as of block 478,558 are also owners of Bitcoin Cash. The new block size allows nodes in the BCH network to process more transactions per second, and to reduce transaction fees.

- *Bitcoin Gold (BTG):* Jack Liao founded Bitcoin Gold on October 24, 2017. The fork occurred at the block 419,406 and as in Bitcoin Cash, the owner of 1 BTC also gets 1 BTG. As expected, Bitcoin Gold reaches consensus using proof-of-work, but the main difference from the Bitcoin protocol is the hash algorithm (Equihash, instead of SHA-256). The cause of this fork was centralization since the Bitcoin community perceived mining to be under the control of a few mining pools.

[8]In this sense, Hayes (2017) made an attempt to try to link these factors and found that the cost of mining influences the price of crypto-assets.

[9]In this context, copy means using almost the same consensus algorithm, but with slightly different rules. If the blockchain community wants to switch from proof-of-work to other protocol, for example, proof-of-stake, this is not considered as a fork. At least for the purpose of this paper.

- *Bitcoin Diamond (BCD):* On November 11, 2017, a fork of Bitcoin happened at block 495,866 and as a result, Bitcoin Diamond was created. This fork seeks to add more transaction capacity to the network and lower transaction fees by slightly changing the proof-of-work algorithm. It is not clear what the main difference in the consensus algorithm is. Other differences are the algorithm of the transaction signatures and the money supply (10 times more than BTC).

- *Bitcoin Private (BTCP):* Bitcoin Private is a special crypto-currency since it is a fork from both ZClassic and Bitcoin. In fact, ZClassic itself is a fork of ZCash. Officially, the fork took place at the blocks 511,346 for BTC and 272,991 for ZClassic on February 28, 2018. Bitcoin Private was designed to decrease centralization by increasing the block size to 2MB and reducing the block time to 2.5 minutes.

As a side note, I could have included Bitcoin SV (November 15, 2018) in the sample, however, I was looking for crypto-currencies that had observations longer than a year. In addition, the fork list is large and it is not known with certainty how many coins have originated from Bitcoin technology. As of the time of writing this document, according to CoinDesk and CoinMarketCap, there are at least ten more forks: Bitcoin Atom, Bitcoin Scrypt, Bitcoin Uranium, Bitcoin Rhodium, Bitcoin Energy, Copper Bitcoin, Super Bitcoin, BitCore, Bitcoin Zero, and Bitvolt.

To conclude this section, most people in the crypto-industry agree that the current Bitcoin technology needs to be changed if the entire industry wants to overtake traditional financial institutions. There is an active discussion about increasing the block size and reducing transaction fees, in order to achieve real decentralization and a faster payment system. As Bitcoin is designed, these modifications can be done outside the underlying protocol, i.e., by a fork. But every time there is a fork, the price volatility tends to spike inducing investors to readjust their portfolios. The purpose of the next section is to present the volatility models I use to analyze whether the return on Bitcoin and the return on its forks are related, and how the volatility risk is transmitted among them.

3 The data and risk models

The data set is from coinmarketcap.com which reports historical data for crypto-currency prices since April 28, 2013. The data contain daily closing prices for Bitcoin and Bitcoin forks: Bitcoin and Litecoin (April 2013 - August 2019), Bitcoin Cash (August 2017 - August 2019), Bitcoin Gold (October 2017 - August 2019), Bitcoin Diamond (November 2017 - August 2019), and Bitcoin Private (February 2018 - August 2019).

3.1 Univariate models

To estimate the volatility of crypto-returns, I use three Generalized Autoregressive Conditional Heteroskedastic (GARCH) models. I start with the most standard framework in empirical finance, the GARCH model proposed by Bollerslev (1986). According to Hansen and Lunden (2005), GARCH(1,1) model performs well in most of the cases and the other GARCH specifications do not provide a significant gain in terms of goodness of fit. Nevertheless, I consider the exponential GARCH (EGARCH) model of Nelson (1991), and the threshold GARCH (TGARCH) model of Glosten et al. (1993) to capture a well known feature of financial time series: the leverage effect. That is, volatility tends to respond asymmetrically to "bad news" (excess returns lower than expected) and to "good news" (excess returns higher than expected). Hasen and Lunden (2005) argue that the first order of any of the aforementioned GARCH specifications appears to be adequate to model the volatility of time series.

Let r_t be the logarithmic return on a crypto-currency. The conditional mean and variance of r_t given F_{t-1} are $\mu_t = E(r_t \mid F_{t-1})$ and $\sigma_t^2 = Var(r_t \mid F_{t-1}) = E((r_t - \mu_t)^2 \mid F_{t-1})$, where F_{t-1} denotes the information set available at time $t-1$ and typically consists of all linear functions of the past returns. Let $a_t = r_t - \mu_t$ be the residual of the mean equation or innovation at time t, and $\varepsilon_t = \frac{a_t}{\sigma_t}$ be the standardized residuals which is an independent and identically distributed (i.i.d.) random variable with mean 0 and variance 1.

To fix ideas, the equations for the conditional mean of r_t and the different specifications for modeling the volatility of r_t given F_{t-1}, are presented below.

$$r_t = \mu_t + a_t \tag{1}$$

$$\sigma_t^2 = \omega + \alpha a_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{2}$$

$$\ln(\sigma_t^2) = \omega + +\alpha\left(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|\right) + \gamma\varepsilon_{t-1} + \beta\ln(\sigma_{t-1}^2) \tag{3}$$

$$\sigma_t^2 = \omega + (\alpha + \gamma N_{t-1}) a_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{4}$$

where ε_t is an i.i.d. standard normal process, $E(|\varepsilon_t|) = \sqrt{2/\pi}$, and ω, α, β, and γ are constants. Equation (1) describes the conditional mean of r_t which can be a constant or an ARMA model. For most asset returns the serial correlation is weak and a simple AR model might be enough (Zivot and Wang, 2003).

Equations (2), (3), and (4) refer to the structure of the volatility model under GARCH(1,1), EGARCH(1,1) and TGARCH(1,1) specifications, respectively. Note the standard restrictions on the parameters: $\omega \geqq 0$, $\alpha \geqq 0$, $\beta \geqq 0$ and $\alpha + \beta < 1$; the latter restriction implies that the variance is finite and not integrated. The benefit of equation (3) is that there is no need for further non-negative restrictions for the parameters. A positive a_{t-1} contributes $\alpha(1+\gamma)\,|\varepsilon_{t-1}|$ to the log volatility whereas a negative value of a_{t-1} increases the log volatility in $\alpha(1-\gamma)\,|\varepsilon_{t-1}|$ due to a value of $\gamma < 0$ is expected. Regarding equation (4), the coefficient γ has to be positive in order to capture the leverage effect where $N_{t-i} = 1$ if $a_{t-i} < 0$, and $N_{t-i} = 0$ otherwise.

3.2 Multivariate models

I consider an approach to multivariate volatility modeling using the Exponentially Weighted Moving Average (EWMA), the BEKK-GARCH model of Engel and Kroner (1995), and the DCC-GARCH model of Engel (2002). All the models are generalizations of univariate GARCH models, and allow us to study the dynamic relationship between volatility processes of multiple asset returns. Consider a k-dimensional return series $\{r_t\}$

$$r_t = \mu_t + a_t \tag{5}$$

where $a_t = (a_{1t,}..., a_{kt})'$ is the shock, or innovation, at time t. The mean equation μ_t could follow a multivariate linear model like a VARMA(p,q) structure, or multivariate nonlinear models. As Tsay (2010) argues, it is enough to employ a simple VARMA structure with exogenous variables.

The conditional covariance matrix of a_t given F_{t-1} is a $k \times k$ positive-definite matrix $\{\Sigma_t\}$ defined by $\Sigma_t = Cov(a_t \mid F_{t-1})$. The shock can be written as $a_t = \Sigma_t^{1/2}\epsilon_t$ where $\Sigma_t^{1/2}$ is the square-root matrix of Σ_t, and ϵ_t is a sequence of i.i.d. random vector such that $E(\epsilon_t) = 0$ and $Cov(\epsilon_t) = I_k$.

A simple multivariate volatility framework is the EWMA model. Let $\widehat{a}_t$ be the residuals of the mean equation. The model for the conditional covariance matrix is

$$\widehat{\Sigma}_t = \lambda\widehat{\Sigma}_{t-1} + (1-\lambda)\widehat{a}_{t-1}\widehat{a}'_{t-1} \tag{6}$$

where $0 < \lambda < 1$ denotes the decaying rate or the persistent parameter. For a given λ and initial estimate of covariance matrix ($\widehat{\Sigma}_0$), $\widehat{\Sigma}_t$ can be computed recursively. A common choice of $\widehat{\Sigma}_0$ is the sample covariance matrix of $\widehat{a}_t$ which produces a positive-definite volatility matrix ($\widehat{\Sigma}_t$) for all t. A major drawback with the EWMA model is that it tends to reject the diagnostic tests in empirical applications.

The second multivariate volatility method is the BEKK-GARCH model which is concerned with the dynamic evolution of Σ_t. For a k-dimensional time series r_t, the BEKK-GARCH(1,1) specification assumes the form

$$\Sigma_t = A_0A'_0 + A_1a_{t-1}a'_{t-1}A'_1 + B_1\Sigma_{t-1}B'_1 \tag{7}$$

where A_0 is a lower triangular matrix such that is $A_0A'_0$ a posititve-definite matrix, and A_1 and B_1 are $k \times k$ matrices. The main advantage of this approach is that it allows dependence between the volatility series. In addition, the model solves the problem of positive-definite constraint, i.e., it provides positive-definite volatility matrix Σ_t for all t. On the other side, the main disadvantage of this model is that it contains too many parameters, $k^2 + [k(k+1)/2]$. Also, the parameters in A_1 and B_1 do not have a direct interpretation.

Finally, the dynamic conditional correlation (DCC) model is built on the idea of modeling the conditional variances and correlations instead of modeling Σ_t. Let the conditional correlations to be time-varying

$$\rho_t = D_t^{-1}\Sigma_t D_t^{-1} \tag{8}$$

where $D_t = diag\left\{\sigma_{11t}^{1/2}, ..., \sigma_{kkt}^{1/2}\right\}$ is the diagonal matrix of the k volatilities at time t, and ρ_t is the correlation matrix with $k(k-1)/2$ elements. To fit a DCC-GARCH model, we first estimate individually each element of D_t (σ_{iit}) using any univariate GARCH specification and form estimated standardized residuals, and then model the pairwise conditional correlations between the standardized residuals.

In this regard, Engel (2002) proposes the following correlation structure

$$Q_t = (1 - \theta_1 - \theta_2)Q + \theta_1 Q_{t-1} + \theta_2 \epsilon_{t-1}\epsilon'_{t-1} \tag{9}$$

where Q_t is the covariance matrix of standardized residuals, Q is the unconditional covariance matrix of standardized residuals, and θ_1 and θ_2 are non-negative real numbers satisfying $0 < \theta_1 + \theta_2 < 1$. The correlation matrix is defined as $\rho_t = J_t Q_t J_t$, where $J_t = diag\left\{q_{11t}^{-1/2}, ..., q_{kkt}^{-1/2}\right\}$ is a normalization matrix and q_{iit} denotes the (i,i)th element of Q_t. The parameters θ_1 and θ_2 describe the dynamic dependence of the correlation matrix, and make the DCC model very parsimonious. But it is hard to justify that all correlations evolve in the same manner regardless of the assets involved (Bauwens et al., 2006).

4 Results

I investigate the interdependencies in risk-return between the return on Bitcoin and the returns on its forks: Litecoin (LTC), Bitcoin Cash (BCH), Bitcoin Gold (BTG), Bitcoin Diamond (BTD), and Bitcoin Private (BTCP). This set of five forks are particularly interesting for examining the relationship between a specific consensus protocol (Bitcoin proof-of-work) and its forks.

With the exception of BTCP, all other crypto-returns have a weak serial correlation based on ACF and PACF. Also I estimate AR(p) model for each return series, and only BTCP and BTD need an AR(1) specification[10]. The mean equation indicates that the lagged daily log returns of crypto-currencies are not relevant factors of current log returns, at least lagged values greater than one period. This result is in line with the empirical fact of a low serial correlation of asset returns (Zivot and Wang, 2003). Also, the first-order autoregressive process could be a sign of violation of the efficient market hypothesis. By building a model for the conditional mean, I eliminate any linear dependence. However, the returns on tokens can still be serially non-linear dependent due to ARCH effects.

Table 1 shows the Ljung-Box test results for the innovations of log returns. One interesting finding is that the null hypothesis is rejected at 5% for lag values longer than 10. Except for Litecoin, there is no serial correlation at very short intervals of time. On the other hand, the LM test for ARCH effects reveals that the null hypothesis is rejected for all the time series and at different lag values. Therefore, the squared residuals are positively correlated even though the innovations themselves are not.

After testing for the presence of ARCH effects, the next step is to specify a univariate GARCH model for the crypto-returns. Table 2 gives the results for GARCH(1,1), EGARCH(1,1), and TGARCH(1,1). The three estimated models are essentially the same and produce a high persistence of the variance, especially for

[10]The results are available upon request.

BTG, BTD and BTCP. Based on the GARCH(1,1), we can compute the half-life defined as the number of days it takes for half of the expected reversion back towards the long-run variance: BTC (25.32 days), LTC (22 days), BCH (4.39 days), BTG (86.29 days), BTD (40.43 days), and BTCP (692.8 days). It is clear that BTCP has a strong volatility persistence.

From Table 2, it can be noticed that the Akaike Information Criterion (AIC) is minimized and the log-likelihood function is maximized under the TGARCH(1,1) model in most of the cases. Nevertheless, the difference in these information criteria between the EGARCH(1,1) and TGARCH(1,1) models is marginal. As a side note, the AR(1)-TGARCH(1,1) specification better fits the volatility dynamics for Bitcoin Diamond and Bitcoin Private. Moreover, all the estimated parameters are statistically significant and point in the right direction. But, contrary to expectations, the EGARCH(1,1) produces a positive leverage parameter which is difficult to explain. If we focus our attention only on Bitcoin (first column of Table 2), my findings are different from Bouoiyour and Selmi (2015) and Katsiampa (2017).

If AIC and log-likelihood function are used in model selection, one selects the TGARCH(1,1) specification. Therefore, I proceed to do the model checking. Table 3 presents the p-value of the Weighted Ljung-Box test and the Weighted ARCH LM test on standardized squared residuals. Both tests fail to reject the null hypothesis at different lags, indicating that the model fits the data well and captures all ARCH effects.

Figure 3 gives the fitted volatility series of the TGARCH(1,1) model. From this figure, we can be see that there are no jumps in the volatility of any of the return series, the volatility does not diverge to infinity, and the leverage effect is present. As expected, the volatility was high from the beginning of 2017 until mid-2018. It seems that the Bitcoin bubble in 2017 contributes significantly to the high volatility persistence of Bitcoin forks. The only crypto-currency that exhibits higher volatility in the second half of the sample (since November 2018) is Bitcoin Private.

Once the volatility model is selected, I carry out a time-varying correlation analysis in order to get a better understanding of the sign and the strength of the dynamic correlation between the return on Bitcoin and the returns on its forks. The correlation between Bitcoin and Litecoin exhibits an interesting pattern. For the first part of the sample, the correlation was strong and positive. In fact, the highest positive correlation was in December 2013. After that, it decreased steadily and remained low until the beginning of the year 2017. During the bubble period, the correlation between these returns strengthened, increasing from about 0 in March 2017, to 0.7 on February 2018. Subsequently, it went back to the pre-bubble levels.

The most striking finding is related to Bitcoin Cash, the most popular Bitcoin fork. For the second half of the year 2017, the correlation between Bitcoin and Bitcoin Cash was negative, and they had a perfect negative correlation on December 21, 2017. Thereafter, the correlation stabilized around 0.2.

The results of the other three crypto-currency returns are remarkable too. To begin with, the correlation of Bitcoin Gold with Bitcoin presents an expected dynamic. During the first two months after BTG's inception in October 2017, the statistical association between these two returns was negative. From that moment on, the correlation oscillated between 0 and 0.2. The story of Bitcoin Diamond is quite different. Its association with Bitcoin was unstable during the November 2017 - August 2018 period, sometimes positive and sometimes negative. Afterwards, the correlation decreased to a low positive value. Finally, Bitcoin Private was not strongly correlated with Bitcoin until April 2019. In the next month, the correlation plunged to a negative value (-0.8), and two months later it reached its peak (1.0).

Turning now to the multivariate volatility approach, I use the EWMA, BEKK-GARCH(1,1) and DCC-GARCH(1,1) specifications to examine whether these models produce results similar to those of the TGARCH(1,1) model. Before applying the multivariate method, it is necessary to check for the presence of conditional heteroscedasticity in the two dimensional time series. It should be noted that each pair of returns has different number of observations. Thus, the tests are carried out based on the return with fewer observations. Table 4 displays the results of four tests for detecting conditional heteroscedasticity in the bivariate innovations. As we expect, the test statistics reject the null hypothesis confirming the presence of conditional heteroscedasticity.

Figure 5 provides time plots of the volatilities of the EWMA (blue line) and BEKK-GARCH(1,1) (black line) models. As expected, the estimated volatilities series produced by the EWMA approach are smoother than the volatilities series produced by the BEKK model. Nevertheless, the two volatilities series share a similar pattern. What is surprising is that the estimated volatilities of BTC, LTC, and BTCP produced by the two multivariate methods are very close to the univariate results. On the other side, the volatilities of BCH, BTG, and BCD produced by the multivariate models are slightly higher than the volatilities of the TGARCH(1,1) specification. It is important to bear in mind that I used the standardized residuals of the TGARCH(1,1) estimation to fit the DCC-GARCH(1,1) model. Thus, the DCC does not estimate the volatility return.

Equally significantly, the parameter λ that governs the time dynamics of the EWMA covariance matrix lies in the typical range commonly seen in practice. The estimated lambda is around 0.97 and is statistically significant at the 1% level: BTC-LTC (0.973), BTC-BCH (0.968), BTC-BTD (0.977), BTC-BTCP (0.970), and BTC-BTG (0.975).

On the question of the time-varying correlation, the benefits of applying multivariate volatility models are significant. Figures 6, 7, and 8 present the results of the estimated time-varying correlation by EWMA, BEKK, and DCC models, respectively. Contrary to the univariate case, the multivariate volatility methods produce high and positive correlations between the return on Bitcoin and the returns on its forks, for most of the sample. These figures are revealing in three ways. First, Bitcoin and its forks are negatively correlated during the bubble year. In particular, LTC, BCH, BTG, and BTD are negatively correlated with Bitcoin during the last two months of 2017, while the correlation of LTC with Bitcoin was negative in March, August, November, and December of the same year. Second, due to the fact that LTC is the only fork that happened before the bubble period, we can see that from May 2013 to December 2013, the correlation decreased from a high to a low value, and was even negative in September and October of the same year. Thus, the 2017 correlation resembles the 2013 correlation. Third, BTCP, is highly correlated with Bitcoin, averaging 0.82

until November 2018, and 0.55 thereafter. The decreasing correlation over time is apparent, contrary to the increasing estimated correlation by the TGARCH(1,1) model.

Finally, to check the adequacy of the fitted multivariate volatility models, I employ two portmanteau test statistics: the multivariate Ljung-Box test and the multivariate Lagrange Multiplier (LM) test[11]. As expected, the standardized residuals of the EWMA model still have conditional heteroscedasticity, because both tests reject the null hypothesis (Table 5). Conversely, both statistics fail to reject the null hypothesis of serial correlation for the BEKK and DCC specifications.

4.1 Discussion

The empirical analysis of univariate GARCH models for the return on Bitcoin and the returns on its forks offers a starting point to study the volatility risk in the crypto-market and how this kind of risk is transmitted from a Bitcoin fork to Bitcoin. In terms of volatility, the gains of using a multivariate volatility approach are not substantial. A closer inspection of Figures 3 and 5 reveals that volatility changes are closely linked across crypto-currencies within the crypto-market.

However, the three multivariate volatility models offer a better estimation of the time-varying correlation than the TGARCH(1,1) results. All the models produce higher time-varying correlations between Bitcoin and each of its forks, and the overall pattern of the time-varying correlations based on the EWMA, BEKK, and DCC specifications seem similar to each other. In particular, the BEKK method appears to have stronger persistence in the time-varying relationship between crypto-currencies.

I think that one of the most important findings that emerges from the time-varying correlation analysis is the negative value during times of high risk (November 2017 – December 2017), but positive value in times of low risk (May 2018 – July 2019). An implication of this is the possibility that Bitcoin is not only the leader of the crypto-market but also the most liquid token. In times of turmoil, investors prefer to invest in this crypto-currency rather than in Bitcoin forks. But the positive value for most of the sample suggests that Bitcoin and its forks behave like assets instead of currencies. Therefore, it is not possible to reduce the volatility risk of Bitcoin by taking opposite positions in one of its forks simultaneously. In other words, Bitcoin forks run on top of Bitcoin protocol and these tokens share the same risk as Bitcoin and consequently, Bitcoin forks do not provide benefits of diversification.

Putting all the time-varying correlation results together, Bitcoin consensus protocol and its forks are dynamically related. In fact, Litecoin – a crypto-currency that has been around longer than the other Bitcoin forks – is not more interconnected.

Another remarkable finding is that the 2017 bubble makes more persistent volatility series, and this outcome is regardless of the method that was applied. Moreover, all crypto-currencies display the highest volatility in December 2017. I want to highlight the fact that the highest transaction fees, the highest BCH

[11] For further details, see Tse, Y. K. (2002). Residual-based diagnostics for conditional heteroscedasticity models. The Econometrics Journal 5 (2), 358–374.

volatility, and the perfect negative correlation of BCH with BTC took place on December 21, 2017; 5 days after the highest Bitcoin price, and 1 day after the maximum Bitcoin Cash price.

Perhaps, Bitcoin Cash is the most popular and controversial Bitcoin fork. The relevance of BCH lies not only in the perfect negative correlation with Bitcoin in December 2017, but also in the shape of its volatility risk in 2018. In this regard, BCH risk increased between July and December 2018, and it could be explained by the Bitcoin SV fork. In early 2018, the Bitcoin Cash community was involved in a big disagreement related to its block size. In May 2018, the block size was increased from 8MB to 32MB. On August 16, 2018, a fork of BCH happened after Jimmy Nguyen, the former CEO of nChain, proposed Bitcoin SV as a new chain. In opposition, some members of the Bitcoin Cash community announced Bitcoin ABC as the real blockchain, which was very similar to BCH. Finally, on November 15, 2018, BCH forked into Bitcoin SV and Bitcoin ABC. For a period, both blockchains complete to take control of Bitcoin Cash. According to the Bitcoin Cash website, Bitcoin ABC took over theBitcoin Cash chain and Bitcoin SV is listed as its own coin.

The relationship between BTC and LTC needs special attention during two periods of turmoil. The first period, October 2013-May 2014, was characterized by a high volatility and a decreasing correlation. One factor that could explain these findings is the collapse of Mt. Gox – the biggest Bitcoin exchange market – in February 2014. During this month, Mt. Gox suspended services and filed for bankruptcy protection from creditors, and two months later it liquidation proceedings. Likewise, the second period – the 2017 bubble – exhibits a high volatility but a low and even negative correlation. What is interesting is that the pattern of the time-varying correlation was qualitatively the same during these two episodes of high volatility risk.

Regarding Bitcoin return, it is important to note that my findings are different from Bouoiyour and Selmi (2015)'s and Katsiampa (2017)'s outcomes, because the AR(1) coefficient of the mean equation is absent in my estimates. It could suggest that, at the beginning, the crypto-market was Bitcoin and it was inefficient. But the crypto-market is becoming more efficient over time (Urquhart 2016, and Bariviera 2017). In this line of reasoning, the sample period is relevant because I extend it until August 2019. While I include the 2017 bubble period, Bouoiyour and Selmi study Bitcoin volatility until June 2015 and Katsiampa's estimates are until July 2016.

Last but not least, I show that there is a serial dependence in the crypto-currency returns. However, during the modeling process, removing serial dependence by fitting an ARMA(p,q) model to the mean equation has hardly any effects on the estimated time-varying correlation. As a matter of fact, the time-varying correlations before and after fitting an AR(1) model for Bitcoin Diamond and Bitcoin Private are qualitatively the same. With respect to the volatility equation, any unusual volatility in the innovation (a_t) tends to persist, though not forever. The conditional variance tends to revert to its long-term value, so that the process is stationary with a finite variance.

5 Conclusions

The present study was designed to examine the transmission of the volatility risk from Bitcoin forks to Bitcoin. I provide evidence that the volatility of Bitcoin forks and the volatility of Bitcoin are dynamically related. BEKK produces a more accurate estimation of both the volatilities and the correlations. In particular, the BEKK results suggest that feedback between the volatility series has to be considered in future research.

The fact that the time-varying correlation is negative in times of high risk but positive in times of low risk has an important implication. Bitcoin and its forks behave like currencies (there is substitution among tokens) during episodes of turmoil, but behave like assets during calmer times. This shift from a negative to a positive correlation could induce a readjustment in investors' portfolio causing fluctuations in Bitcoin fork prices.

The time horizon is not relevant for the statistical association between Bitcoin and its forks because Litecoin is not more interconnected than the other forks. After the bubble period, Bitcoin and its forks are strongly positive correlated indicating that investors cannot reduce Bitcoin risk by taking opposite positions in Bitcoin forks.

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Table 1: The p-values of the Ljung-Box test and the Lagrange Multiplier test for detecting conditional heteroscedasticity in crypto-currency returns.

	BTC	LTC	BCH	BCD	BTCP	BTG
Ljung-Box						
$Q(1)$	0.602	0.085	0.442	0.894	0.978	0.442
$Q(2)$	0.253	0.038	0.497	0.246	0.999	0.682
$Q(5)$	0.183	0.007	0.558	0.146	0.368	0.036
$Q(10)$	0.004	0.000	0.519	0.023	0.474	0.071
$Q(15)$	0.027	0.000	0.503	0.030	0.132	0.219
Lagrange Multiplier						
$LM(1)$	0.000	0.000	0.000	0.000	0.000	0.000
$LM(2)$	0.000	0.000	0.000	0.000	0.000	0.000
$LM(5)$	0.000	0.000	0.000	0.000	0.000	0.000
$LM(10)$	0.000	0.000	0.000	0.000	0.000	0.000
$LM(15)$	0.000	0.000	0.000	0.000	0.000	0.000

Q(m) and LM(m) denote the Ljung-Box test and the Lagrande Multiplier test on the innovations of crypto-currency returns at lag m, respectively.

Table 2: Estimation results of GARCH-type models for crypto-currency returns.

	BTC	LTC	BCH	BCD	BTCP	BTG
GARCH(1,1)						
$Const(\mu)$	0.112*	-0.072	-0.160	-0.371	-0.953**	-0.394*
$AR(1)(\phi)$	-	-	-	-0.110***	-0.0118	-
$Const(\omega)$	0.538***	1.054***	7.268	1.280***	0.898*	0.227***
$ARCH(\alpha)$	0.111***	0.084***	0.100***	0.057***	0.069***	0.021**
$GARCH(\beta)$	0.862***	0.885***	0.754***	0.926***	0.930***	0.971***
LL	-6289.97	-7068.24	-2572.63	-2291.8	-2090.72	-2162.81
AIC	5.435	6.107	6.701	7.122	7.776	6.410
EGARCH(1,1)						
$Const(\mu)$	0.131*	-0.024	-0.068	-0.330	-1.577***	-0.346
$AR(1)(\phi)$	-	-	-	-0.103***	-0.019	-
$Const(\omega)$	0.191***	0.188***	0.342***	0.230***	0.877*	0.009
$ARCH(\alpha)$	0.250***	0.174***	0.138***	0.245***	0.195***	0.039***
$GARCH(\beta)$	0.939***	0.951***	0.914***	0.954***	0.986***	0.990***
$Leverage(\gamma)$	-0.001	0.020***	0.041***	0.045***	-0.082***	0.017***
LL	-6276.14	-7071.43	-2573.31	-2286.71	-2083.19	-2163.79
AIC	5.424	6.111	6.704	7.110	7.752	6.4163
TGARCH(1,1)						
$Const(\mu)$	0.122*	0.057	-0.122	-0.363	-1.164***	-0.381*
$AR(1)(\phi)$	-	-	-	-0.108**	-0.021	-
$Const(\omega)$	0.530***	1.054***	5.463	1.395**	0.929*	0.063
$ARCH(\alpha)$	0.117***	0.090***	0.100***	0.066***	0.023***	0.02***
$GARCH(\beta)$	0.863***	0.885***	0.807***	0.921***	0.928***	0.983***
$Leverage(\gamma)$	-0.012	-0.014	-0.037	-0.010	0.097***	-0.017
LL	-6289.72	-7067.72	-2571.98	-2291.72	-2081.29	-2161.42
AIC	5.536	6.108	6.702	7.124	7.745	6.410

(*) Represents the significance at the 10% level, (**) represents the significance at the 5% level, and (***) represents the significance at the 1% level.

Table 3: Model checking of TGARCH specifications - The p-values of Ljung-Box and Lagrange Multiplier tests.

	BTC	LTC	BCH	BCD	BTCP	BTG
Ljung-Box						
$Q(1)$	0.931	0.629	0.250	0.054	0.021	0.991
$Q(5)$	0.806	0.951	0.661	0.070	0.096	0.814
$Q(10)$	0.865	0.989	0.518	0.163	0.207	0.749
Lagrange Multiplier						
$LM(1)$	0.376	0.850	0.793	0.771	0.507	0.468
$LM(5)$	0.378	0.919	0.751	0.821	0.738	0.832
$LM(10)$	0.559	0.969	0.380	0.927	0.873	0.682

Q(m) and LM(m) denote the Weighted Ljung-Box Test and the Weighted ARCH LM Test on standardized squared residuals at lag m, respectively.

Table 4: The p-values of the multivariate Ljung-Box test and the multivariate Lagrange Multiplier test for detecting the conditional heteroscedasticity in a vector of two cryptocurrency returns.

	BTC-LTC	BTC-BCH	BTC-BCD	BTC-BTCP	BTC-BTG
Ljung-Box					
$Q(1)$	0.000	0.000	0.000	0.000	0.000
$Q(5)$	0.000	0.000	0.000	0.000	0.000
$Q(10)$	0.000	0.000	0.000	0.000	
Lagrange Multiplier					
$LM(1)$	0.000	0.000	0.000	0.000	0.000
$LM(5)$	0.000	0.000	0.000	0.000	0.000
$LM(10)$	0.000	0.000	0.000	0.000	0.000

Q(m) and LM(m) denote the Weighted Ljung-Box Test and the Weighted ARCH LM Test on standardized squared residuals at lag m, respectively.

Table 5: Model checking of the multivariate volatility models - The p-values of the multivariate Ljung-Box and the multivariate Lagrange Multiplier tests.

	BTC-LTC	BTC-BCH	BTC-BCD	BTC-BTCP	BTC-BTG
EWMA					
$Q(5)$	0.002	0.007	0.000	0.004	0.000
$Q(10)$	0.046	0.000	0.006	0.000	0.000
$LM(5)$	0.001	0.000	0.001	0.000	0.000
$LM(10)$	0.007	0.000	0.020	0.000	0.000
BEKK-GARCH(1,1)					
$Q(5)$	0.983	-	-	0.000	-
$Q(10)$	0.995	-	-	0.000	-
$LM(5)$	0.828	0.207	0.986	0.000	0.085
$LM(10)$	0.798	0.430	0.643	0.000	0.069
DCC-GARCH(1,1)					
$Q(5)$	0.534	0.003	0.497	0.488	0.000
$Q(10)$	0.621	0.080	0.841	0.011	0.000
$LM(5)$	0.338	0.001	0.329	0.633	0.050
$LM(10)$	0.436	0.001	0.644	0.432	0.084

Q(m) and LM(m) denote the Weighted Ljung-Box Test and the Weighted ARCH LM Test on standardized squared residuals at lag m, respectively.

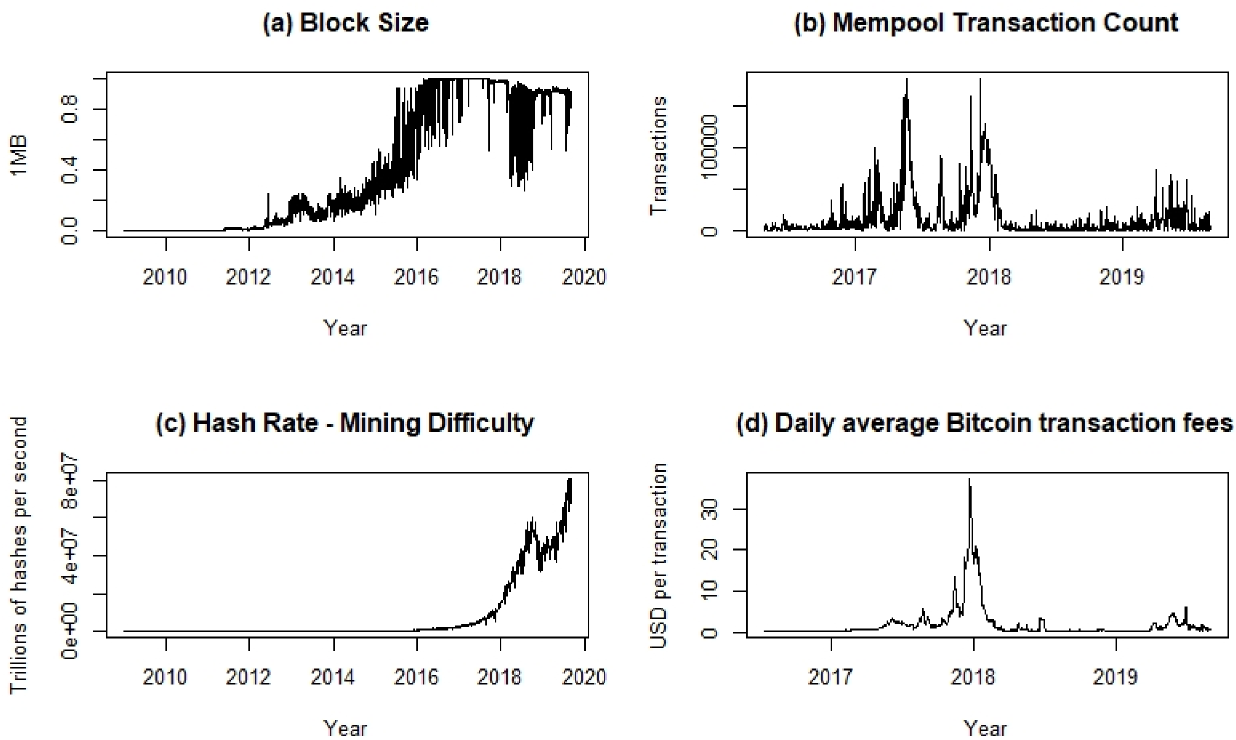

Figure 1: Factors behind a Bitcoin fork. Block Size: data sent to legacy nodes. Mempool: number of unconfirmed transactions that have been broadcast to the Bitcoin network. Hash rate: he estimated number of tera hashes per second (trillions of hashes per second) the Bitcoin network is performing. Bitcoin transaction fees: daily average fees in USD per transaction.
Sources: Block Size - https://bitcoinvisuals.com, Mempool - https://www.blockchain.com, Transaction Fees - https://bitcoinfees.info, and Hash Rate and Mining Difficulty - https://www.blockchain.com.

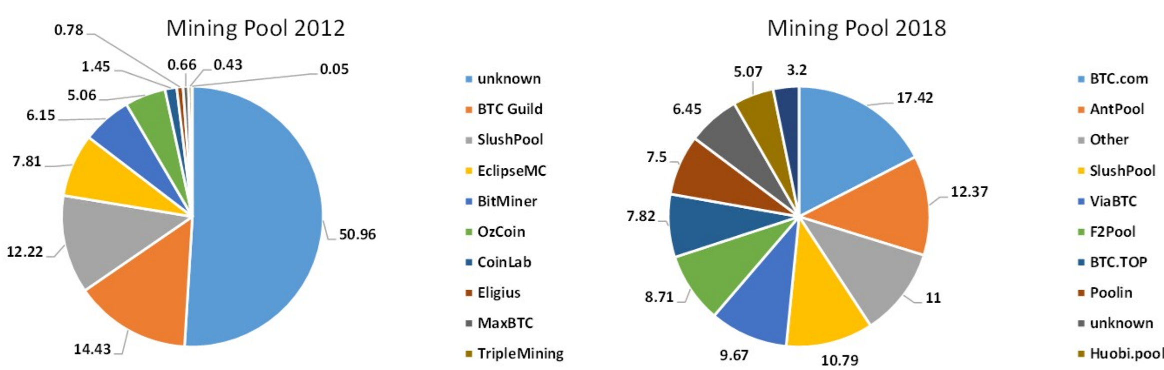

Figure 2: Mining Pool Centralization.

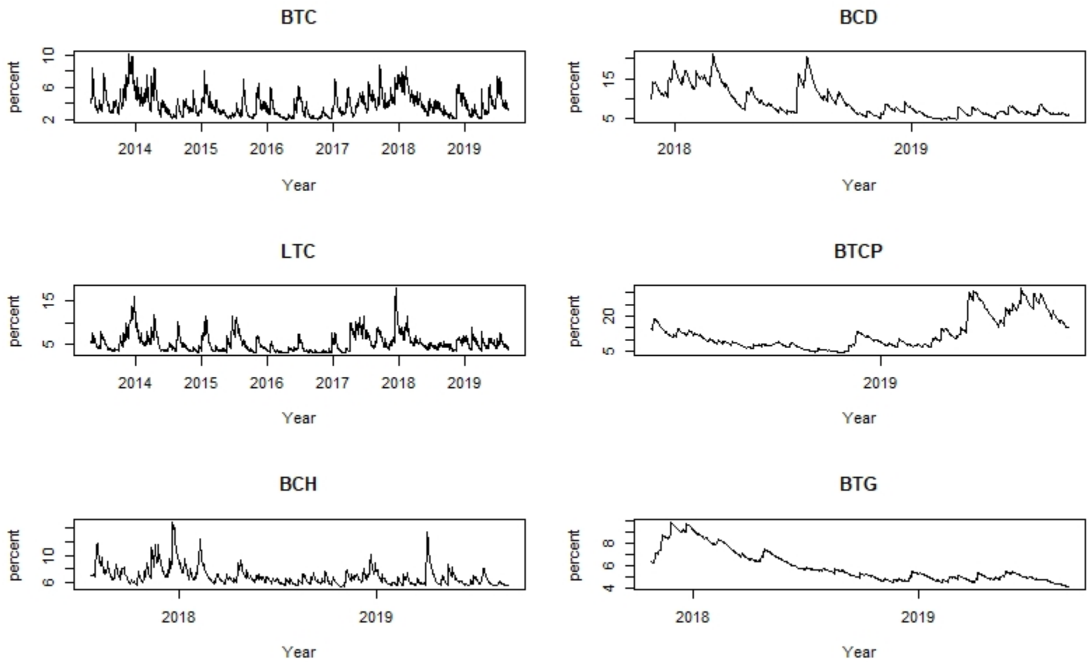

Figure 3: TGARCH(1,1) daily volatility of returns on crypto-currencies.

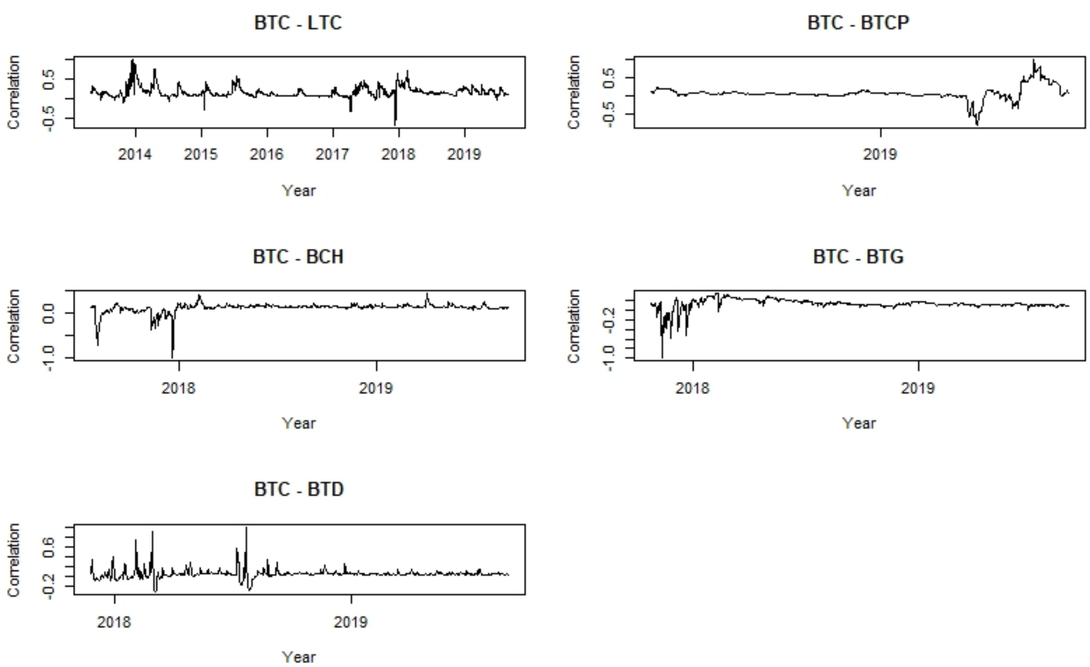

Figure 4: TGARCH(1,1) daily correlation between return on Bitcoin and returns on Bitcoin forks.

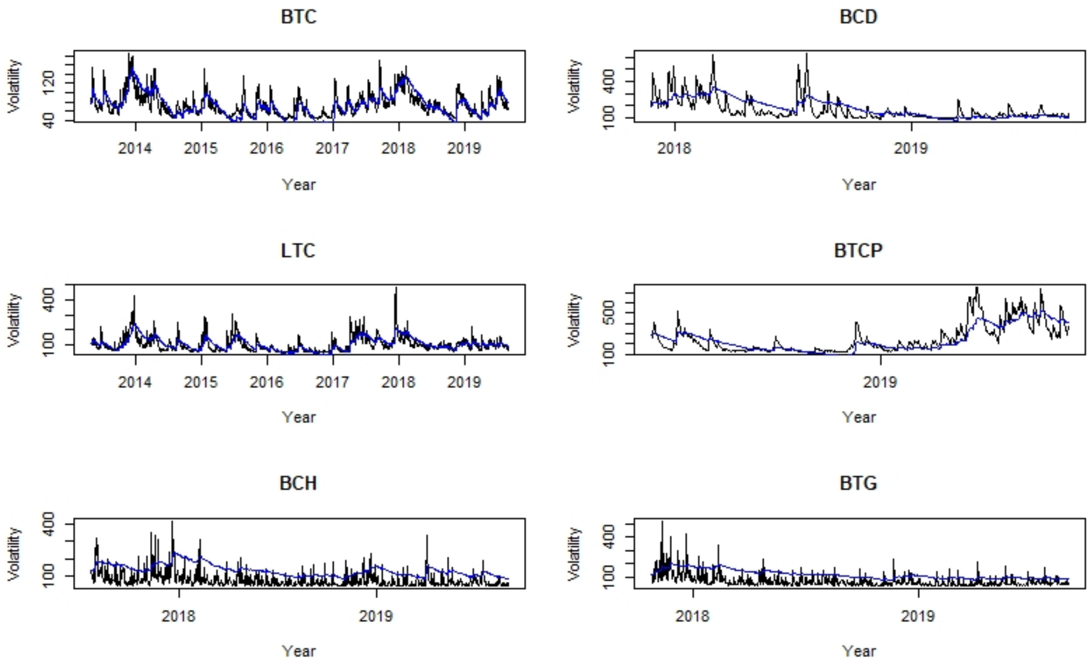

Figure 5: Multivariate daily volatility of returns on crypto-currencies. The blue line is the EWMA estimated volatility and the black line is the BEKK(1,1) estimated volatility.

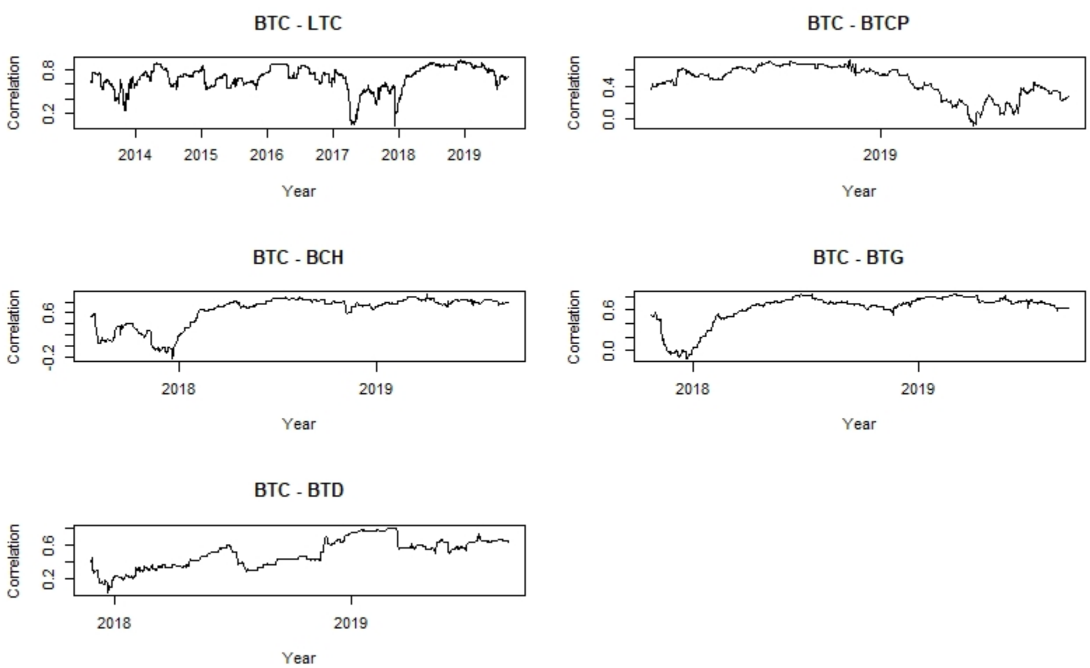

Figure 6: EWMA estimated correlations.

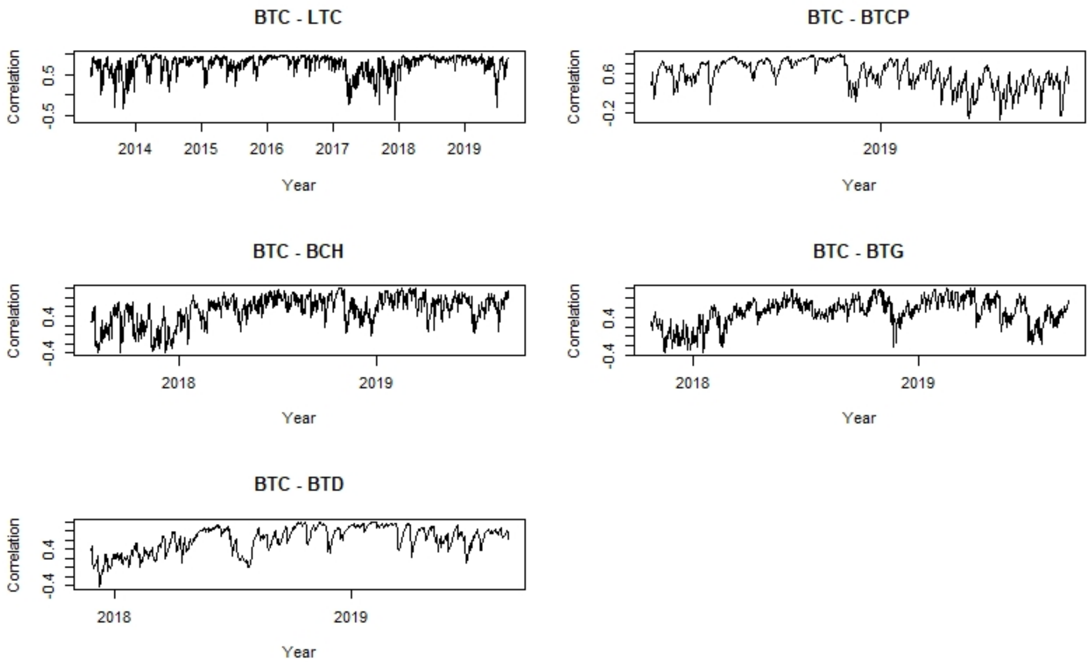

Figure 7: BEKK(1,1) estimated correlations.

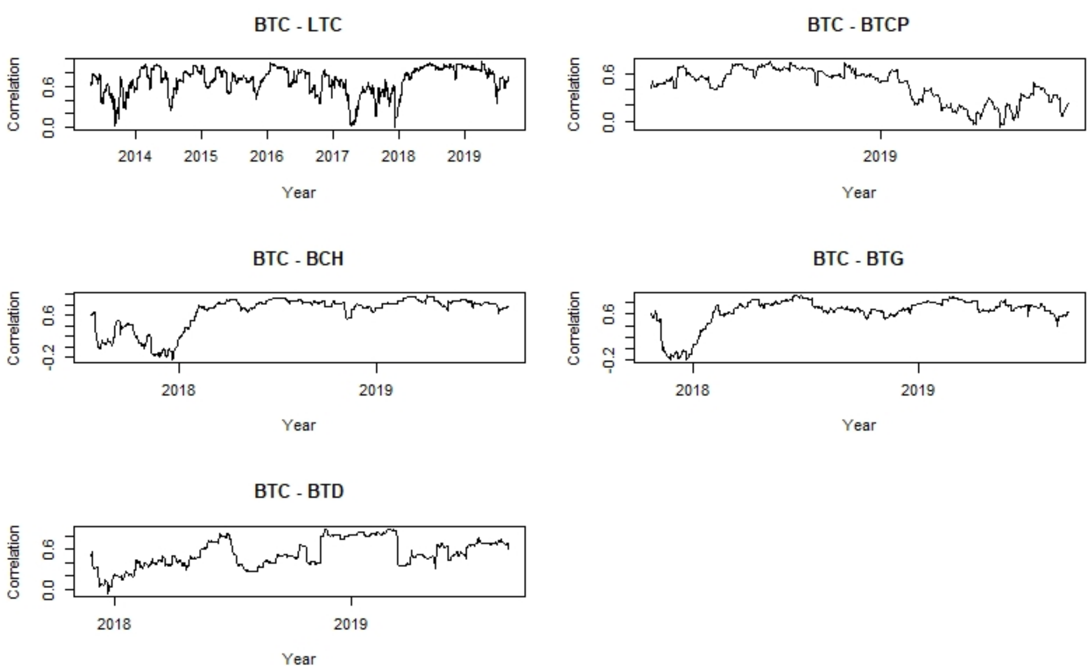

Figure 8: DCC(1,1) estimated correlations.